

Bosonization in SU(N) Gauge Field Theory in Terms of Phase Transition of Second Kind

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Bosonization of the strong interacting matter as a process of arising observable hadrons is studied in terms of the phase transition of the second kind. The spectrum of bosons which is free from the zero point energy is derived. The calculated boson mass is found to depend self-consistently on both the amplitude of a gauge field and quark mass. In the framework of the quasi-classical model[17, 18] a hadron mass is calculated in the case of bosonization into pions.

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I. INTRODUCTION

Since observable particles are colorless the process of the hadronization plays a key role in QCD. The problem complicates when hadrons are mesons, since such particles are governed by the Bose-Einstein statistics while they consist of interacting fermions.

Bozonization generally means a description of fermionic systems in terms of the collective boson degrees of freedom. This concerns both the quantum field theory[1, 2] and condensed matter[3, 4]. It is preferable to bosonate a fermionic system by solving the motion equation for interacting fermions. Although this way is the most correct and elegant it has been still made in the case of the Minkowski space-time $(1 + 1)$. Following such technique the bosonization of the strong interaction matter is considered many times[5–16]. The heart of the method developed in the papers[5–11] is existence of the so-called flux tube, when the strong interacting matter is suggested to be in the condition of the longitudinal dominance and transverse confinement in the Minkowskii space-time. In such consideration the oscillations of color density are governed by the Klein-Gordon equation which mass term contains a boson mass in an explicit form[5–13]. Various mechanisms of the $QCD4 \rightarrow QCD2 \times QCD2$ fragmentation (or the process of arising a quark-gluon tube) which is required in the framework of the consideration developed in Ref.[5–13], are studied in Ref.[9–11]. Separating the longitudinal and transverse motions, the self-consistent set of equations for gauge and fermion fields has been derived[9–11] in the $(1 + 1)$ Minkowskii space-time.

The bosonization is considered in terms of the holographic description of hadrons in string theory[14, 15]. Following the gauge-string duality[16] boson masses have been calculated[14, 15] due to the $(4 \leftrightarrow 10)$ duality.

In the present paper bosonization as a process of arising observable hadrons is studied in terms of the QCD lagrangian in the standard $(1 + 3)$ Minkowski space-time without any prior fragmentation. Considering the bosonization as the equilibrium phase transition of the second kind, the boson spectrum is derived in the quasi-classical approximation beyond the fluctuation region of the transition. The obtained spectrum is free from the zero point energy. When the confinement phase is governed by the equations of the self-consistent quasi-classical model[17, 18] a boson mass is calculated provided that a quark-gluon plasma is bosonated into the lightest bosons (pions).

The paper is organized as follows. The second section contains the general equations governing the confinement phase and the relations corresponding to the main approximations. Bosonization as the phase transition of the second kind is considered in Section III. The bosonization in the framework of the quasi-classical self-consistent model[17, 18] is studied in Section IV. The applicability of the obtained results for describing observable hadrons is discussed in Section V. Appendix I contains the main equations of the quasi-classical self-consistent model[17, 18].

II. GENERAL EQUATIONS

The gauge invariant action \mathcal{A} in the SU(N) field theory is[19–21]:

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$$\mathcal{A} = \int d^4x \left\{ \frac{1}{2} [\bar{\psi}(x) \gamma^k (i\partial_k + gT_a A_k^a) \psi(x) - \bar{\psi}(x) m \psi(x)] - \frac{1}{2} [\bar{\psi}(x) \gamma^k (i\overleftarrow{\partial}_k - gT_a A_k^a) \psi(x) + \bar{\psi}(x) m \psi(x)] - \frac{1}{16\pi} F_{\mu\nu}^a F_a^{\mu\nu} \right\}, \quad (1)$$

where $F_{\mu\nu}^a$ is the tensor of the non-abelian gauge field which is given by the expression:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{bc}^a A_\mu^b A_\nu^c \quad (2)$$

The action (1) generates the energy-momentum tensor

$$T^{\mu\nu} = \frac{i}{2} \left\{ \bar{\Psi}(x) \gamma^\mu \partial^\nu \Psi(x) - \bar{\Psi}(x) \gamma^\mu \overleftarrow{\partial}^\nu \Psi(x) \right\} + g (J^a(x))^\mu A_\mu^\nu(x) + \frac{1}{4\pi} \left\{ -F_a^{\mu i}(x) (F_a^\nu)_i(x) + \frac{\mathcal{G}^{\mu\nu}}{4} F_a^{ik}(x) F_{ik}^a(x) \right\} \quad (3)$$

and the motion equations:

$$\partial_\mu F_a^{\nu\mu}(x) - g \cdot f_{ab}^c A_\mu^b(x) F_c^{\nu\mu}(x) = -g J_a^\nu(x) \quad (4)$$

$$F_a^{\nu\mu}(x) = \partial^\nu A_a^\mu(x) - \partial^\mu A_a^\nu(x) - g \cdot f_a^{bc} A_b^\nu(x) A_c^\mu(x), \quad (5)$$

$$J_a^\nu(x) = \bar{\Psi}(x) \gamma^\nu T_a \Psi(x), \quad (6)$$

In this way, the fermion fields $\Psi(x), \bar{\Psi}(x)$ are governed by the Dirac equation:

$$\{i\gamma^\mu (\partial_\mu + ig \cdot A_\mu^a(x) T_a) - m\} \Psi(x) = 0 \quad (7)$$

$$\bar{\Psi}(x) \left\{ i\gamma^\mu \left(\overleftarrow{\partial}_\mu - ig \cdot A_\mu^{*a}(x) T_a \right) + m \right\} = 0; \quad . \quad (8)$$

In Eqs.(1) - (8) we introduce the following notations; m is a fermion mass, g is the coupling constant; γ^ν are the Dirac matrixes, $x \equiv x^\mu = (x^0; \vec{x})$ is a vector in the Minkowski space-time; $\partial_\mu = (\partial/\partial t; \nabla)$; the Roman letters numerate a basis in the space of the associated representation of the $SU(N)$ group, so that $a, b, c = 1 \dots N^2 - 1$. We use the signature $diag(\mathcal{G}^{\mu\nu}) = (1; -1; -1; -1)$ for the metric tensor $\mathcal{G}^{\mu\nu}$. The line over Ψ mean the Dirac conjugation. Summing over any pair of the repeated indexes is implied.

The symbols T_a in Eqs.(1)-(8) are the generators of the $SU(N)$ group which satisfy the commutative relations and normalization condition:

$$[T_a, T_b]_- = T_a T_b - T_b T_a = i f_{ab}^c T_c; \quad f_{ab}^c = -2 i Tr([T_a, T_b]_- T_c) \quad (9)$$

$$Tr(T_a T_b) = \frac{1}{2} \delta_{ab}; \quad (10)$$

where f_{ab}^c are the structure constants of the $SU(N)$ group, which are real and anti-symmetrical with respect to the transposition in any pair of indexes; δ_{ab} is the Kroneker symbol.

We assume that the field $A_\nu^a(x)$ depends on coordinates via some scalar function $\varphi(x)$ in the Minkowski space-time which is normally named by the eikonal:

$$A_\nu^a(x) = A_\nu^a(\varphi(x)). \quad (11)$$

Let the axial gauge be for the field $A_\mu^a(x)$:

$$\partial^\mu A_\mu^a = 0; \quad k^\mu \dot{A}_\mu^a = 0, \quad (12)$$

where the dot over the letter means differentiation with respect to the introduced variable φ while the vector k^μ is:

$$k^\mu = \partial^\mu \varphi(x) \quad (13)$$

The introduced vector k^μ indicates the direction along the eikonal, while Eq.(12) means the local transversion of the gauge field A_μ^a .

Due to Eq.(11) the hamiltonian generated by the action (1) does not depend explicitly on the time variably. This means that some stationary states of fermions, which energy is $\varepsilon(\vec{p})$, exist.

Then, the hamiltonian of interacting fermions can be written as follows:

$$\mathcal{H} = \int d^3\vec{x} T^{00} = \sum_{\vec{p}; \sigma, \alpha} \varepsilon(\vec{p}) [n_{\sigma, \alpha}(\vec{p}) + (1 - \bar{n}_{\sigma, \alpha}(\vec{p}))] + \int d^3\vec{x} T_g^{00}, \quad (14)$$

where T_g^{00} is the zeroth component of the momentum-energy tensor of the gauge field; $n_{\sigma, \alpha}(\vec{p})$ and $\bar{n}_{\sigma, \alpha}(\vec{p})$ are the occupation numbers of fermions and anti-fermions, respectively.

III. BOSONIZATION

In studying bosonization we follow the assumption that bosonization starts when the fermion vacuum is full such that the occupation number of both particles $n_{\sigma, \alpha}(\vec{p})$ and anti-particles $\bar{n}_{\sigma, \alpha}(\vec{p})$ are equal to unit:

$$n_{\sigma, \alpha}(\vec{p}) = 1; \quad \bar{n}_{\sigma, \alpha}(\vec{p}) = 1. \quad (15)$$

As for the a gluon field, we assume that the number of gluons is large $n_g \gg 1$ due to the self-interaction of them. Since $n_g \gg 1$, the gluon field can be considered quasi-classically.

Besides that we suggest that fermions and gauge field are in equilibrium.

On the other hand, A_μ^a is self-interacting field that leads to the generation of new quanta of A_μ^a in spite of the equilibrium state. Then, the creation of additional quanta on the background of the fullness of the fermion vacuums has to result in arising new particles since the entropy is in maximum.

Let us consider the matter consisting of interacting quarks and gluons. We assume that the matter transits to the deconfinement phase so that bosons only arise as observable particles.

We present the gauge field as a sum of two orthogonal terms[22–24] in the group space such that A_μ^a has the following form:

$$A_\mu^a(\varphi) = \mathcal{A}_\mu^a + e_\mu^a \Phi(\varphi); \quad \mathcal{A}_\mu^a e_b^\mu = 0; \quad e_\mu^a e_b^\mu = -\delta_b^a, \quad (16)$$

where \mathcal{A}_μ^a is amplitude of the gauge field just before the phase transition. The amplitude \mathcal{A}_μ^a is taken to be constant in the Minkowski space-time, while $\Phi(\varphi)$ is a scalar function therein. The field $\Phi(\varphi)$ is not to equal to zero in the deconfinement phase, and plays a role of the order parameter. We note that the presentation of \mathcal{A}_μ^a in the form given by Eq.(17) means that the phase transition is considered beyond the fluctuation region[25].

Then, the gluon part of the momentum-energy tensor T_g^{00} , which is given by Eq.(14), is of the form:

$$T_g^{00} = \frac{1}{16\pi} \left\{ 4(N^2 - 1) (\partial^0 \Phi)^2 - 2(N^2 - 1) (\partial^\nu \Phi) (\partial_\nu \Phi) + 2Ng^2 A^2 \Phi^2 + g^2 f_a^{bc} f_{b_1 c_1}^a \mathcal{A}_b^\nu \mathcal{A}_c^\mu \mathcal{A}_\nu^{b_1} \mathcal{A}_\mu^{c_1} + \right. \\ \left. g^2 N(N^2 - 1) \Phi^4 \right\}; \quad -A^2 \equiv \mathcal{A}_\mu^a \mathcal{A}_a^\mu \quad (17)$$

We study the situation when the density n_0 of the particles governed by the field Φ is not too large, so that

$$n_0^{1/3} \lambda_C \ll 1, \quad (18)$$

where $\lambda_C = 1/M$ is the Compton wave length of a particle, which mass is M . Such inequality corresponds to studying the phase transition beyond the fluctuation region[?].

Then, the last term in Eq.(18) is small[29]. As a result, taking into account Eq.(15), we rewrite the hamiltonian given by Eq.(14) by the following way:

$$\mathcal{H} = \sum_{\vec{p}; \sigma, \alpha} \varepsilon(\vec{p}) + \frac{1}{16\pi} \int d^3 \vec{x} \left\{ 4(N^2 - 1) (\partial^0 \Phi)^2 - 2(N^2 - 1) (\partial^\nu \Phi) (\partial_\nu \Phi) + 2Ng^2 A^2 \Phi^2 + \right. \\ \left. g^2 f_a^{bc} f_{b_1 c_1}^a \mathcal{A}_b^\nu \mathcal{A}_c^\mu \mathcal{A}_\nu^{b_1} \mathcal{A}_\mu^{c_1} \right\}. \quad (19)$$

We should note here that the hamiltonian (20) is independent on the color variables in the explicit form.

By changing $\Phi \rightarrow \vec{\Phi}$, the relations (20) is easy generalized to the case when the field $\vec{\Phi}$ is the triplet of pseudoscalar mesons, where $\vec{\Phi}$ is the vector in the isospace.

We expand $\vec{\Phi}(\varphi)$ over the whole set of the plane waves:

$$\vec{\Phi}(\varphi) = \sum_{\vec{q}} \sqrt{\frac{8\pi}{V(N^2 - 1) \omega(\vec{q})}} \left\{ \vec{e} c(\vec{q}) \exp(-iqx) + \vec{e}^* c^\dagger(\vec{q}) \exp(iqx) \right\}, \\ \omega(\vec{q}) = \sqrt{\vec{q}^2 + M^2}, \quad M^2 = \frac{Ng^2 A^2}{(N^2 - 1)}; \quad \vec{e} \vec{e}^* = 1, \quad (20)$$

where \vec{e} is the unit vector in the isospace; $c(\vec{q})$ and $c^\dagger(\vec{q})$ are the operators of annihilation and creation of the on-shell particle ($q^2 = M^2$) with the 4-momentum $q = (\omega(\vec{q}); \vec{q})$. The operators $c(\vec{q})$ and $c^\dagger(\vec{q})$ satisfy the standard Bose-Einstein commutative relations.

Let us substitute the expansion given by Eq.(21) into the formula (20) and average the obtained relation over the vacuum of the field $\vec{\Phi}$. As a result, we derive the energy of the particles governed by the pseudoscalar field $\vec{\Phi}$:

$$E = \sum_{\vec{q}} \omega(\vec{q}) \langle c^\dagger(\vec{q}) c(\vec{q}) \rangle + \left\{ \frac{1}{2} \sum_{\vec{q}} \omega(\vec{q}) + \sum_{\vec{p}; \sigma, \alpha} \varepsilon(\vec{p}) + g^2 f_a^{bc} f_{b_1 c_1}^a \mathcal{A}_b^\nu \mathcal{A}_c^\mu \mathcal{A}_\nu^{b_1} \mathcal{A}_\mu^{c_1} \right\}, \quad (21)$$

where the angle brackets mean averaging over the pseudoscalar vacuum.

Since the vacuum of arising pseudoscalar particles should be empty the term in the curl brackets has to be equal to zero. This condition determines the spectrum $\varepsilon(\vec{p})$ of quarks via the gauge field \mathcal{A}_μ^c just before the phase transition. We should note here that the last term in the curl bracket should be negative.

As a result, we obtain the energy spectrum of scalar hadrons:

$$E_h = \sum_{\vec{q}} \omega(\vec{q}) N_h(\vec{q}); \quad \omega(\vec{q}) = \sqrt{\vec{q}^2 + M^2} \quad (22)$$

provided that

$$\left\{ \frac{1}{2} \sum_{\vec{q}} \omega(\vec{q}) + \sum_{\vec{p}; \sigma, \alpha} \varepsilon(\vec{p}) + g^2 f_a^{bc} f_{b_1 c_1}^a \mathcal{A}_b^\nu \mathcal{A}_c^\mu \mathcal{A}_\nu^{b_1} \mathcal{A}_\mu^{c_1} \right\} = 0, \quad (23)$$

where $N_h(\vec{q})$ is the number of the on-shell hadrons with the 4-momentum $q = (\omega(\vec{q}); \vec{q})$.

IV. BOSONIZATION IN QUASI-CLASSICAL MODEL

Let us apply the results obtained in the previous sections to the calculation of a boson mass in terms of the self-consistent quasi-classical model developed in Ref.[17, 18] (see, also Appendix):

In this case the convolution in Eq.(22) is equal to[18]

$$-f_a^{bc} f_{b_1 c_1}^a \mathcal{A}_b^\nu \mathcal{A}_c^\mu \mathcal{A}_\nu^{b_1} \mathcal{A}_\mu^{c_1} = (N^2 - 1) \sum_{\sigma\alpha} \int \frac{d^3 p}{p^{(0)}(2\pi)^3}. \quad (24)$$

Then, the boson mass M is given by a formula:

$$M^2 \approx \frac{2 N N_f \alpha_s}{2|C|\pi} Q^2; \\ \alpha_s = \frac{g^2}{4\pi}, \quad C = -f_a^{bc} f_{b_1 c_1}^a \cos(\varphi_b - \varphi_{b_1}) \cos(\varphi_c - \varphi_{c_1}) > 0, \quad (25)$$

where α_s is the strong interaction coupling constant, Q is the transferred momentum corresponding to the confinement-deconfinement phase transition which is of the order of the phase transition temperature. The parameters φ_{b,c,b_1,c_1} are the phases of the amplitudes $\mathcal{A}_{b,c,b_1,c_1}^\nu$ which are fixed such that the convolution C is negative.

In the case $N_f = 2$; $N = 3$, we have:

$$M \approx \sqrt{\frac{12\alpha_s}{\pi}} Q. \quad (26)$$

The last formula establish relation of the hadron mass M to the momentum of interacting particles in the matter which depends strongly on the matter temperature T .

If we set $T = Q = 213 MeV$ [26], then $\alpha_s = 0.12$. As a result we obtain:

$$M \approx 144 MeV \quad ; |C| \sim 1, \quad (27)$$

that corresponds to the pion mass.

Although the derived pion mass is very nearly to the tabulated date Eq.(26) should be mainly treated as the formula giving the relation of a hadron mass to the temperature of phase transition. Particular, when the phase transition temperature is around 200 MeV the result for the mass of observable particles is found to be correct.

In the case $\varepsilon(\vec{p}) = \sqrt{\vec{p}^2 + m^2}$ [17, 18], the condition (24) leads to

$$\frac{V_0}{V} \sim \alpha_s \ll 1, \quad (28)$$

where m is a quark mass; V_0 and V are the volumes occupied by the quark-gluon and hadron phases, respectively. The derived inequality is expectable and means that the volume occupied by hadrons is much greater as compared with one for a quark-gluon plasma.

V. DISCUSSION

We discuss the obtained results in terms of the key assumption consisting in negligibility of the last term in (18). To do it we calculate the contribution of this term, ΔE , into the spectrum (23) when the field $\vec{\Phi}$ is given by Eq.(21).

In this case direct calculations gives:

$$\Delta E \simeq \frac{1}{16\pi} \int d^3 \vec{x} \langle g^2 N(N^2 - 1) \Phi^4 \rangle = \frac{6}{16\pi} g^2 N(N^2 - 1) \int \frac{V^2 d^3 \vec{q}}{(2\pi)^3} \left(\frac{8\pi}{V(N^2 - 1)\omega(\vec{q})} \right)^2 N_h^2(\vec{q}), \quad (29)$$

where $N_h^2(\vec{q})$ is the mean value of the occupation number of hadrons; $\omega(\vec{q})$ is the hadron energy given by Eq.(20), V is the volume occupied by the hadrons. The factor 6 has arisen due to taking into account the transpositions in the operators c^\dagger and c . The angle brackets mean averaging over the hadron vacuum.

When free hadrons are in equilibrium the number of them, N_h , is governed by the Bose-Einstein distribution function with the zeroth potential, $\mu = 0$. Then, we derive from the last formula:

$$\Delta E = \frac{48 \alpha_s N M}{(N^2 - 1)} \int_0^\infty \frac{\xi^2 d\xi}{\xi^2 + 1} \frac{1}{\left(e^{\frac{\sqrt{\xi^2 + 1}}{T/M_h}} - 1\right)^2}, \quad \alpha_s = \frac{g^2}{4\pi}. \quad (30)$$

Calculation of the energy according to the formula (23) results in:

$$E = \frac{VM^4}{2\pi^2} \int_0^\infty \frac{(\xi^2 + 1)^{1/2} \xi^2 d\xi}{\left(e^{\frac{\sqrt{\xi^2 + 1}}{T/M_h}} - 1\right)}. \quad (31)$$

As a result, the correction to the energy (23) due to the Φ^4 -term is:

$$\frac{\Delta E}{E} = \frac{96\pi^2 N \alpha_s}{(N^2 - 1)} \frac{\int_0^\infty (\xi^2 + 1)^{-1} \xi^2 \left(e^{\frac{\sqrt{\xi^2 + 1}}{T/M_h}} - 1\right)^{-3} d\xi}{\int_0^\infty (\xi^2 + 1)^{1/2} \xi^2 \left(e^{\frac{\sqrt{\xi^2 + 1}}{T/M_h}} - 1\right)^{-1} d\xi} \left(\frac{1}{VM^3}\right) \sim n_0 \lambda_C^3; \quad \lambda_C = M^{-1}. \quad (32)$$

It follows from the last formulae that the correction is proportional to the gas parameter $n_0 \lambda_C^3 \ll 1$ which has been already introduced by Eq.(19).

In the RHIC and SPS experiments[26, 27] the characteristic temperature of hadronic phase is $T_h \approx 200 MeV$ while the radius of the fireball is $r_F \geq 10F$. Then, calculating the integrals in Eqs.(31), (33) numerically, we derive:

$$\frac{\Delta E}{E} \lesssim 1 \cdot 10^{-3}, \quad (33)$$

that proves reasonability of the used approximation.

The estimations of the corrections to the spectral distribution of the energy $\delta(\Delta E)/\delta N_h$ can be directly derived from Eqs.(30), (31), and result in:

$$\frac{\delta(\Delta E)}{\omega(\vec{q}) \delta N_h(\vec{q})} = \frac{192\pi^2 N \alpha_s}{(N^2 - 1)} \frac{1}{(q^2/M^2 + 1)^{3/2}} \left(\frac{1}{VM^3}\right) \left(e^{\frac{\sqrt{q^2/M^2 + 1}}{T/M_h}} - 1\right)^{-1} \sim n_0 \lambda_C^3; \quad \lambda_C = M^{-1}. \quad (34)$$

The numerical calculations according to the formula (35) gives that the maximum of the value of $\frac{\delta(\Delta E)}{\omega(\vec{q}) \delta N_h(\vec{q})}$ (when $q = 0$) is no more than 0.24.

The carried out estimation have been made in the case of the hadronization at the temperature $T_c = 200 MeV$ which is likely to be upper magnitude for T_c . In the assumption of the adiabatic model of the expending matter[28], when

$$V T^3 = const, \quad (35)$$

the both Eqs. (33),(35) decrease with temperature decreasing. This means that correction to the energy spectrum due to Φ^4 terms become smaller if the realistic hadronization temperature appears to be less than the considered $T_c = 200 MeV$.

We should point out that the inequality $n_0^{1/3} \lambda_C \ll 1$ and Eq.(29) are not in contradiction. It follows from the relations:

$$n_0 \lambda_C^3 = \frac{n_0}{M^3} \sim \frac{N_h}{M^3 V} \sim \frac{V_0}{V} \ll 1, \quad (36)$$

where N_h is the number of hadrons which mass is M .

The formula (29) can be also treated in terms of the equilibrium phase transition. Since the transition is equilibrium one the entropy is in a maximum. In order the entropy keeps its maximum, when the phase volume of the confinement phase is decreased due to the bosonization, the phase volume of the hadronic phase has to be increased.

We should note here that the relation (26), (28) can be also considered as a way to calculate the strong coupling constant α_s . Provided that a boson mass has been already known due to experiments, Eqs.(25) and (27) allow us to estimate the value of the constant α_s in various energy regions.

VI. CONCLUSION

The bosonization as the phase transition of the second kind is considered in terms of the QCD gauge invariant lagrangian in the standard $(1+3)$ Minkowski space-time. In the quasi-classical approximation the spectrum of bosons which is free from the zero point energy is derived beyond the fluctuation region of the transition[25]. When the confinement phase is governed by the equations of the self-consistent quasi-classical model[17, 18] the boson mass is calculated provided that the bosonization into the lightest bosons (pions) only takes place. The obtained mass are found to correspond quantitatively to the pion mass provided that the phase transition temperature is near $200 MeV$.

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 - [29] The detailed consideration of this fact is given in the section Discussion

Appendix A

The equations (4)-(8) have the self-consistent solutions which can be written as follow.[17, 18] The fermion field is governed by the formula:

$$\begin{aligned}\Psi(x) &= \sum_{\sigma,\alpha} \int \frac{d^3p}{\sqrt{2p^0} (2\pi)^3} \left\{ \hat{a}_{\sigma,\alpha}(\vec{p}) \Psi_{\sigma,\alpha}(x, p) + \hat{b}_{\sigma,\alpha}^\dagger(\vec{p}) \Psi_{-\sigma,\alpha}(x, -p) \right\} \\ \bar{\Psi}(x) &= \sum_{\sigma,\alpha} \int \frac{d^3p}{\sqrt{2p^0} (2\pi)^3} \left\{ \hat{a}_{\sigma,\alpha}^\dagger(\vec{p}) \bar{\Psi}_{\sigma,\alpha}(x, p) + \hat{b}_{\sigma,\alpha}(\vec{p}) \bar{\Psi}_{-\sigma,\alpha}(x, -p) \right\},\end{aligned}\quad (\text{A1})$$

where the symbols $\hat{a}_{\sigma,\alpha}^\dagger(\vec{p}); \hat{b}_{\sigma,\alpha}^\dagger(\vec{p})$ and $\hat{a}_{\sigma,\alpha}(\vec{p}); \hat{b}_{\sigma,\alpha}(\vec{p})$ are the operators of creation and cancellation of a fermion ($\hat{a}_{\sigma,\alpha}(\vec{p}); \hat{a}_{\sigma,\alpha}^\dagger(\vec{p})$) and anti-fermion ($\hat{b}_{\sigma,\alpha}(\vec{p}); \hat{b}_{\sigma,\alpha}^\dagger(\vec{p})$), respectively. In this way, $\hat{a}_{\sigma,\alpha}(\vec{p})$ and $\hat{a}_{\sigma,\alpha}^\dagger(\vec{p})$; $\hat{b}_{\sigma,\alpha}(\vec{p})$ and $\hat{b}_{\sigma,\alpha}^\dagger(\vec{p})$ satisfy the standard commutative relations for the fermion operators.

The function $\Phi_{\sigma,\alpha}(x, p)$ has a form:

$$\begin{aligned}\Psi_{\sigma,\alpha}(x, p) &= \Phi_{\sigma,\alpha}(x, p) = \cos \theta \cdot \exp \left(-ig^2 \frac{(N^2 - 1)A^2}{2N(pk)} \varphi - ipx \right) \left\{ \left(1 - igT_a \frac{\tan \theta}{\theta(pk)} \int_0^\varphi d\varphi' (A_\mu^a p^\mu) \right) + \right. \\ &\quad \left. \frac{g(\gamma^\nu k_\nu)(\gamma^\mu A_\mu^a)}{2(pk)} \cdot \left[\frac{\tan \theta}{\theta} T_a + \frac{g}{(pk)} \frac{1}{2N} \left(-i \frac{\tan \theta}{\theta} + \frac{g}{(pk)} \frac{\theta - \tan \theta}{\theta^3} T_b \int_0^\varphi d\varphi' (A_\mu^b p^\mu) \right) \int_0^\varphi d\varphi' (A_\nu^a p^\nu) \right] \right\} u_\sigma(p) \cdot v_\alpha; \\ \theta &= \frac{g}{(pk)} \sqrt{\frac{1}{2N}} \left(\int_0^\varphi d\varphi' (A_\mu^a(\varphi') p^\mu) \int_0^\varphi d\varphi'' (A_\mu^a(\varphi'') p_\mu) \right)^{\frac{1}{2}}; \quad (\partial_\nu k^\nu) = (\partial_\nu \partial^\nu) \varphi(x) = 0.\end{aligned}\quad (\text{A2})$$

In this way, the spinors $u_\sigma(p)$ satisfy the relations:

$$\sigma^{\mu\nu} k_\mu A_\nu(\varphi = 0) u_\sigma(p) = 0; \quad \bar{u}_\sigma(p) u_\lambda(p') = \pm 2m \delta_{\sigma\lambda} \delta_{pp'}; \quad p^2 = m^2, \quad (\text{A3})$$

where $u_\sigma(p)$ are the bispinors of the free Dirac field. The plus and minus signs in Eq.(A3) correspond to the Dirac scalar production of the spinors $u_\sigma(p)$ and $u_\sigma(-p)$, respectively, while the function $\Psi_{\sigma,\alpha}(x, p)$ are normalized by the condition:

$$\int d^3x \Psi_{\sigma,\alpha}^*(x, p') \Psi_{\sigma,\alpha}(x, p) = (2\pi)^3 \delta^3(\vec{p} - \vec{p}'). \quad (\text{A4})$$

As for the gauge field it is determined by the equations:

$$\begin{aligned}A_a^\nu(\varphi) &= A \left(e_{(1)}^\nu(\varphi) \cos(\varphi(x) + \varphi_a) + e_{(2)}^\nu(\varphi) \sin(\varphi(x) + \varphi_a) \right) + \mathcal{B}_a \partial^\nu \varphi(x) \\ e_{(1)}^\nu e_{(2)\nu} &= e_{(1)}^\nu k_\nu = e_{(2)}^\nu k_\nu = 0; \quad \dot{e}_{(1)}^\nu = e_{(2)}^\nu; \quad \dot{e}_{(2)}^\nu = -e_{(1)}^\nu; \quad k^\nu \equiv \partial^\nu \varphi(x),\end{aligned}\quad (\text{A5})$$

where $e_{(1),(2)}^\nu(\varphi)$ are the space-like 4-vectors on the wave surface $\varphi(x)$ which are independent on the group variable a ; the symbols A , \mathcal{B}_a and φ_a are some constants in the Minkowski space-time. They are determined via the initial condition of the considered problem.

The fermion and gauge fields are found to not be independent and to relate one to another by the formulae:

$$2f_{ab}^c \sin(\varphi_b - \varphi_c) = f_{ab}^c \left\{ f_c^{sr} \cos(\varphi_b - \varphi_r) + \{\cos(\varphi_b - \varphi_r) \cos(\varphi_s - \varphi_a)\} \frac{f_c^{bs}}{N} \right\} \mathcal{B}_s; \quad (\text{A6})$$

$$A^2 \cdot C = -(N^2 - 1) \sum_{\sigma\alpha} \int \frac{d^3p}{p^{(0)}(2\pi)^3} \langle \hat{a}_{\sigma,\alpha}^\dagger(\vec{p}) \hat{a}_{\sigma,\alpha}(\vec{p}) + \hat{b}_{\sigma,\alpha}(\vec{p}) \hat{b}_{\sigma,\alpha}^\dagger(\vec{p}) \rangle, \quad (\text{A7})$$

where

$$C = f_{ab}^c f_c^{sr} \{\cos(\varphi_b - \varphi_r) \cos(\varphi_s - \varphi_a)\} < 0, \quad (\text{A8})$$

$$(\partial_\mu \varphi(x)) \cdot (\partial^\mu \varphi(x)) = 0; \quad (\partial_\mu \partial^\mu) \varphi(x) = 0. \quad (\text{A9})$$